ランダム作用素のスペクトルと関連する話題 Spectra of Random Operators and Related Topics

平成25年度科研費基盤研究(C)「定常点過程論の枠組みによるランダム作用素のスペクトル統計の研究」 (代表者:慶應義塾大学医学部 南 就将)、平成25年度科研費基盤研究(C)「ランダムシュレディンガー 作用素のスペクトルの確率論的研究」(代表者:京都大学大学院人間・環境学研究科 上木直昌)および平 成25年度科研費基盤研究(A)「シュレディンガー方程式のスペクトル・散乱理論の研究」(代表者:東京 大学大学院数理科学研究科 中村 周)による表記の研究集会を下記のように開催します。

日程/Date 平成25年12月5日(木)-7日(土)/December 5-7, 2013

場所/Venue 京都大学 人間・環境学研究科棟 2階226室/ Room 226, Graduate School of Human and Environmenetal Studies Bldg., Kyoto University

組織員会:中村 周(東京大学)、上木 直昌(京都大学)、南 就将(慶應義塾大学) 連絡責任者:南 就将 tel: 045-566-1352 e-mail: minami@a5.keio.jp

> Organizing committee: Shu Nakamura (Univ. ofTokyo) Naomasa Ueki (Kyoto Univ.) Nariyuki Minami (Keio Univ.) Contact to: minami@a5.keio.jp

プログラム/Program

12月5日(木)/Dec. 5

10:00–10:50 Makoto Katori (Chuo Univ.) Determinantal martingales and noncolliding diffusion processes.

11:00–11:50 Sergio Andraus (Univ. Tokyo) Limiting regimes of interacting particle systems using Dunkl operators.

13:30–14:20 Tomi Ohtsuki (Sophia Univ.)Density of states scaling in disordered topological insulators.

14:30–15:20 Tomohiro Sasamoto (Chiba Univ.) Replica and dualities for KPZ systems.

15:40–16:30 Fumihiko Nakano (Gakushuin Univ.) Level statistics for 1-dimensional random Schrödinger operators.

16:40–17:30 Kazuyoshi Yata, Makoto Aoshima (Univ. Tsukuba)Eigenvalue estimation of large dimensional covariance matrices and its applications.

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12月6日(金)/Dec. 6
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- **10:00–10:50** Victor Chulaesky (Univ. Reims) Efficient bounds for *N*-particle Anderson localization in random and deterministic disordered media.
- 11:00–11:50 Hatem Najar Asymptotic behavior of the IDS for a percolation model in the continuum.
- 13:30–14:20 Abel Klein (Univ. California, Irvine) Localization for multi-particle continuous Anderson Hamiltonians.
- 14:30–15:20 Shin'ichi Kotani (Kwansei Gakuin Univ.)A remark on Schrödinger operators with random potentials of Cauchy distribution.
- 15:40–16:10 Nariyuki Minami (Keio Univ.) Outline of the proof of Oseledec' theorem.
- 16:20–17:10 Keith Slevin (Osaka Univ.) The transfer matrix methods and Lyapunov exponents in Anderson localization.

17:10- Discussion

12月7日(土)/Dec.7

- 10:00–10:50 Takuya Mine (Kyoto Inst. Tech.)Depletion of states for the Hamiltonian with two magnetic vortices.
- 11:00–11:50 Tohru Koma (Gakushuin Univ) A Hall conductance formula for lattice models.
- 13:30-14:20 Taro Nagao (Nagoya Univ.)Products of truncated unitary random matrices.

14:30–15:20 Nariyuki Minami (Keio Univ.)

Definition and self-adjointness of the stochastic Airy operator.

(2013年11月10日現在/Nov. 10, 2013)

Abstracts

Sergio Andraus: We examine the extension of Dyson's Brownian motion and the Wishart-Laguerre processes to real positive values of the parameter beta. These can be formulated as systems of Brownian motions and Bessel processes which repel mutually through a logarit hmic potential; we refer to these systems as interacting Brownian motions and interacting Bessel processes. At the same time, one may use Dunkl operators to define analogs of multidimensional Brownian motion, known as Dunkl processes. The symmetric component of these processes is equivalent to interacting Brownian motions for the Dunkl processes of type A, and to the in teracting Bessel processes for the Dunkl processes of type B. Using analytical and numerical techniques, we investigate the form of these systems' time-scaled particle distributions in the steady state. We show that they correspond to the eigenvalue distribution of the beta-Hermite and beta-Laguerre ensembles of random matrices for arbitrary rapidly-decreasing initial d istributions, and we give estimates of their relaxation time. Additionally, we prove that in the freezing limit (when beta t ends to infinity), the particle distribution of both systems becomes a sum of delta functions located at the zeroes of the H ermite and Laguerre polynomials for the interacting Brownian motion and Bessel processes, respectively. An important part of our analysis is focused on Dunkl's intertwining operators. These operators are constructed to map regular partial derivatives into Dunkl operators, but their explicit general form is unknown. We obtain our results by deriving previously unknown ex pressions for the intertwining operators of type A and B.

[1] SA, M. Katori, S. Miyashita, J. Phys. A 45 395201 (2012) [2] SA, M. Katori, S. Miyashita, arXiv:1309.2733

Hatem Najar: We study Lifshitz tails for continuous Laplacian in a continuous site percolation situation. By this we mean that we delete a random set Γ_{ω} from \mathbf{R}^d and consider the Dirichlet or Neumann Laplacian on $D = \mathbf{R}^d \setminus \Gamma_{\omega}$. We prove that the integrated density of states exhibits Lifshitz behavior at the bottom of the spectrum when we consider Dirichlet boundary conditions, while when we consider Neumann boundary conditions, it is bounded from below by a van Hove behavior. The Lifshitz tails are proven independently of the percolation probability, whereas for the van Hove case we need some assumption on the volume of the sets taken out as well as on the percolation probability. It is a joint work with W. Kirsch (FernUni Hagen). Tomi Ohtsuki: The history of topological insulators (TI) dates back to early 1980's when the quantum Hall effect was discovered. Recent discoveries of two dimensional quantum spin Hall states and three dimensional TIs have inspired extensive research for these novel materials. In the impurity free systems where the translational invariance exists, the topological insulator is characterized by the non-zero topological numbers, which are defined via integral over Brillouin zone. This definition becomes meaningless once the translational invariance is broken due to disorder. In this case, we usually use edge/surface states to characterize TIs. Here we study the bulk properties of the disordered three dimensional topological insulators numerically, and show how to distinguish TI from ordinary insulators by investigating bulk states. We first calculate bulk conductance from which we draw the phase diagram for disordered TI. We then show that the density of states exhibits novel scaling behavior along the phase boundary of different TI phases, which is related to the renormalization of the Dirac electron velocity.

References:

 K.-I. Imura et al.: Physical Review B 86, 245436 (2012) 2) K. Kobayashi, T. Ohtsuki, K.-I. Imura: PRL 110, 236803 (2013) 3) K. Kobayashi, T. Ohtsuki, K.-I. Imura, I. Herbut: arXiv:1308.3953 4) S. Matsuo, et al.: Physical Review B 88, 155438 (2013)

LOCALIZATION FOR MULTI-PARTICLE CONTINUOUS ANDERSON HAMILTONIANS

ABEL KLEIN UNIVERSITY OF CALIFORNIA, IRVINE

We present an extension of the bootstrap multiscale analysis to multi-particle continuous Anderson Hamiltonians, obtaining Anderson localization with finite multiplicity of eigenvalues, a strong form of dynamical localization, and decay of eigenfunction correlations. (Joint work with Son Nguyen.)

References

Abel Klein and Son T. Nguyen: Bootstrap multiscale analysis and localization for multi-particle continuous Anderson Hamiltonians. Preprint.