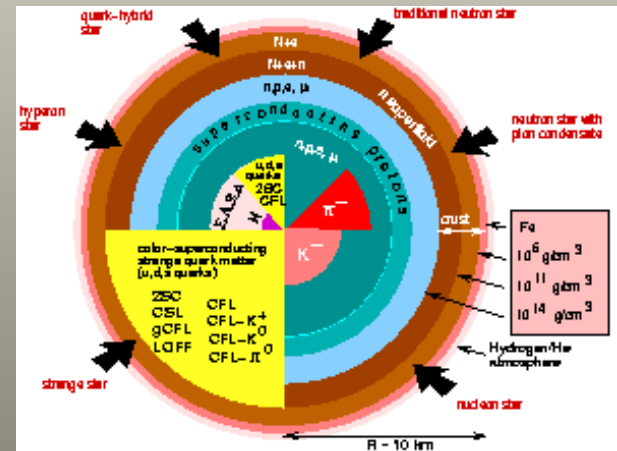


Part 2. Charm quark in quark matter

Content of Part 2

1. Heavy quark in quark matter
2. “Kondo phase” of heavy quark matter
S.Y., K. Suzuki, K. Itakura, arXiv:1604.09229 [hep-ph]
3. Case of single heavy quark
S.Y., arXiv:1608.06450 [hep-ph]



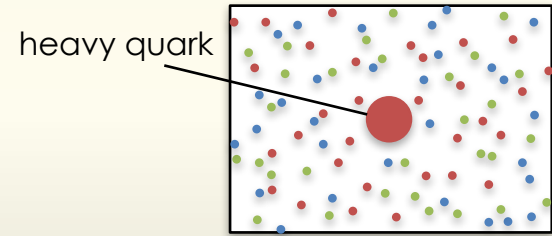
2. “Kondo phase” of heavy quark matter

Lagrangian

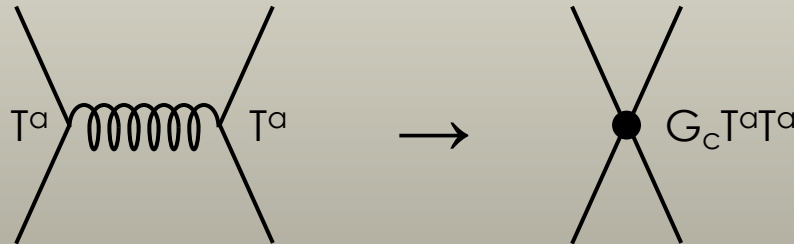
Light quark (u,d,s) ψ and heavy quark (c) Ψ

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \mu \bar{\psi} \gamma^0 \psi + \bar{\Psi} i \not{\partial} \Psi - m_Q \bar{\Psi} \Psi \\ - G_c \sum_a (\bar{\psi} \gamma^\mu T^a \psi) (\bar{\Psi} \gamma_\mu T^a \Psi) ,$$

Cf. NJL for light quark
 $G_c = (9/2) 2.0 / \Lambda^2$
 $\Lambda = 0.65 \text{ [GeV]}$



① Color-current interaction



Cf. S.Y., K. Sudoh, PRC88, 015201 (2013)

K. Hattori, K. Itakura, S. Ozaki, S.Y., PRD92, 065003 (2015)

② N_f flavor for light quark

$$\psi = (\psi^1, \dots, \psi^{N_f})^t$$

③ Heavy quark limit for charm quark $p^\mu = m_Q v^\mu + k^\mu \quad v^\mu v_\mu = 1$

$$\Psi \rightarrow \Psi_v = \frac{1}{2}(1 + \not{v}) e^{im_Q v \cdot x} \Psi$$

← Heavy Quark Effective Theory

Cf. “Heavy Quark Physics”,

V. A. Manohar and M. B. Wise

2. “Kondo phase” of heavy quark matter

Mean-field approximation

S.Y., K. Suzuki, K. Itakura,
arXiv:1604.09229 [hep-ph]

① Mean-field: mixing between light quark and heavy quark

$$(\bar{\psi}_\alpha^i \Psi_{v\delta})(\bar{\Psi}_{v\gamma} \psi_\beta^i) \rightarrow \langle \bar{\psi}_\alpha^i \Psi_{v\delta} \rangle \bar{\Psi}_{v\gamma} \psi_\beta^i + \langle \bar{\Psi}_{v\gamma} \psi_\beta^i \rangle \bar{\psi}_\alpha^i \Psi_{v\delta} - \langle \bar{\psi}_\alpha^i \Psi_{v\delta} \rangle \langle \bar{\Psi}_{v\gamma} \psi_\beta^i \rangle$$

i=light flavor; $\alpha, \beta, \gamma, \delta$ =Dirac indices



② Gap (color singlet)

$$\Delta_{\delta\alpha}^i \equiv \frac{G_c}{2} \langle \bar{\psi}_\alpha^i \Psi_{v\delta} \rangle \rightarrow \Delta_{\delta\alpha}^i = \Delta^i \left(\frac{1+\gamma_0}{2} (1 - \hat{k} \cdot \vec{\gamma}) \right)_{\delta\alpha}$$



③ Mean-field Lagrangian

Lagrange multiplier for heavy quark density n_Q

$$\begin{aligned} \mathcal{L}^{\text{MF}} = & \bar{\psi}(\not{k} + \mu \gamma^0)\psi + \bar{\Psi}_v v \cdot k \Psi_v - \lambda (\Psi_v^\dagger \Psi_v - n_Q) \\ & + \sum_i \Delta^i \bar{\Psi}_v \frac{1+\gamma_0}{2} (1 + \hat{k} \cdot \vec{\gamma}) \psi^i \\ & + \sum_i \Delta^{i*} \bar{\psi}^i (1 + \hat{k} \cdot \vec{\gamma}) \frac{1+\gamma_0}{2} \Psi_v - \sum_i \frac{8}{G_c} |\Delta^i|^2 \end{aligned}$$

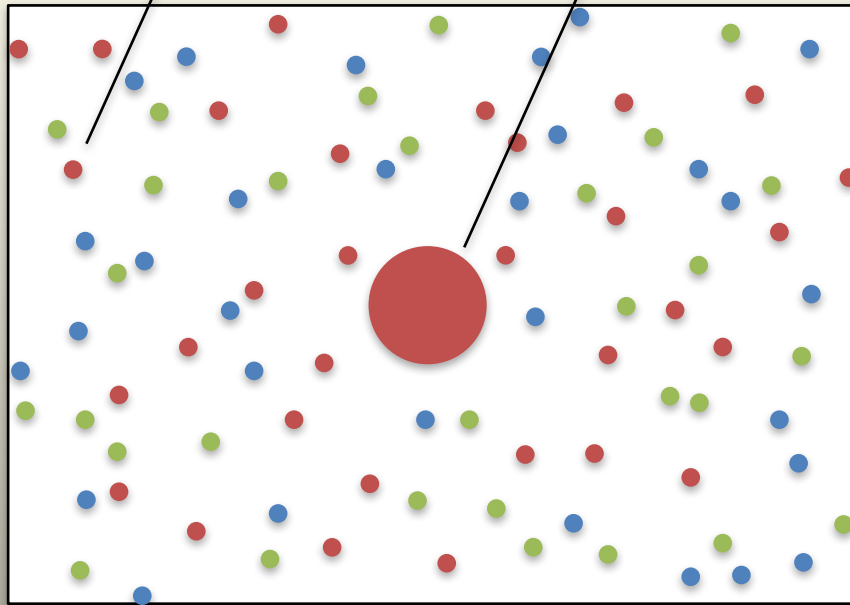
2. “Kondo phase” of heavy quark matter

Charm quark density $n_Q (\lambda)$

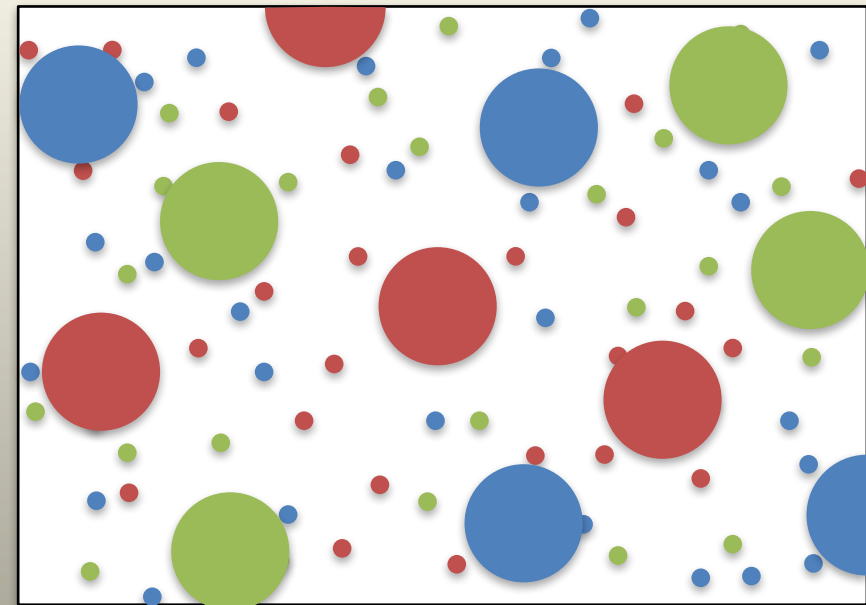
S.Y., K. Suzuki, K. Itakura,
arXiv:1604.09229 [hep-ph]

light quark gas

charm quark



single charm quark



many charm quarks distributed randomly
(No Fermi surface)

Breaking of translational invariance...

S.Y., arXiv:1608.06450 [hep-ph]

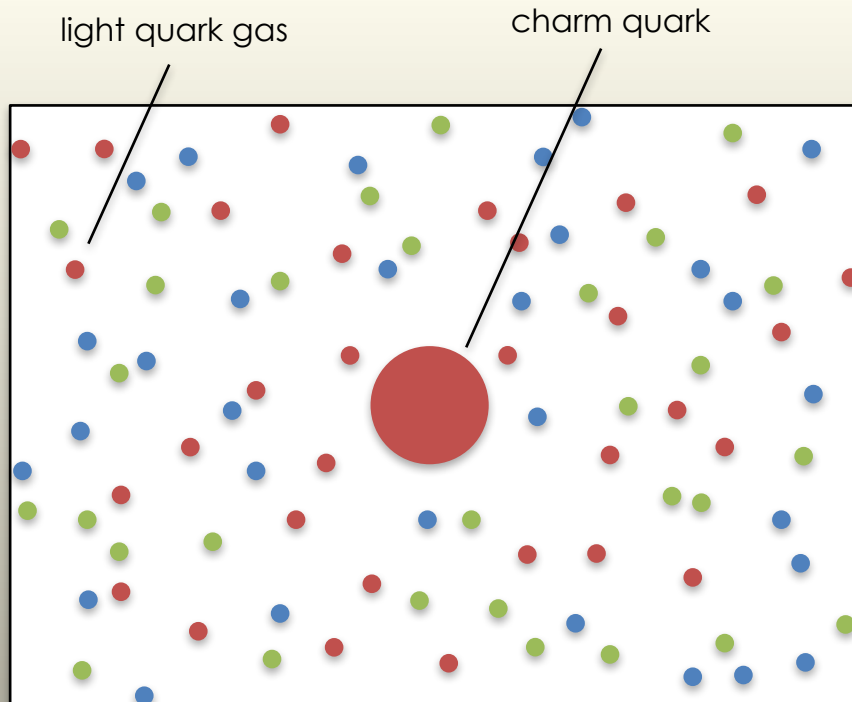
Conservation of translational invariance!

$$\Psi_v^\dagger(x) \Psi_v(x) = \sum_i \delta^{(3)}(\vec{x} - \vec{x}_i)$$

2. “Kondo phase” of heavy quark matter

S.Y., K. Suzuki, K. Itakura,
arXiv:1604.09229 [hep-ph]

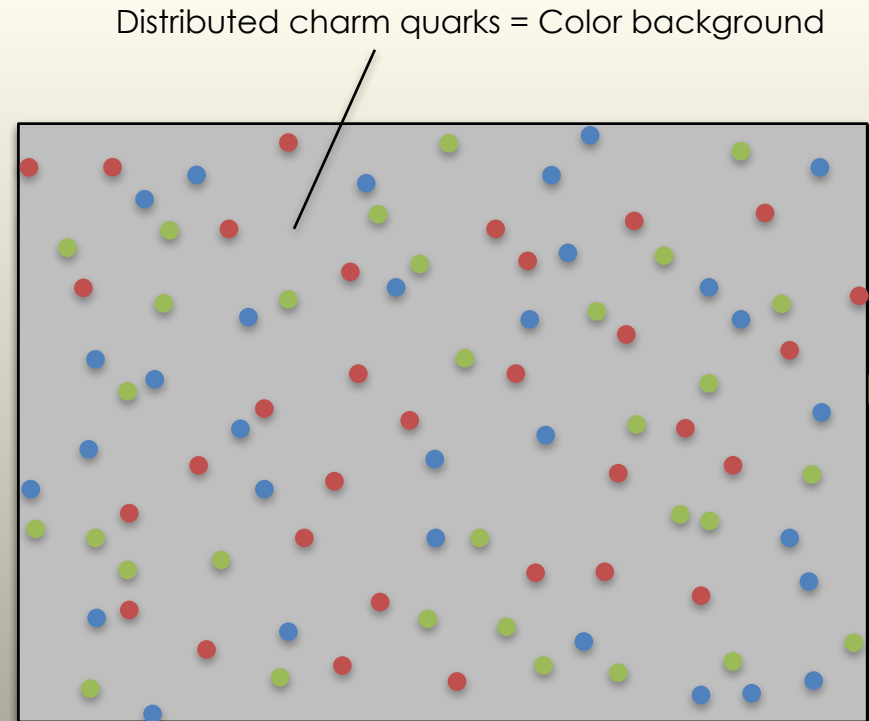
Charm quark density $n_Q (\lambda)$



single charm quark

Breaking of translational invariance...

S.Y., arXiv:1608.06450 [hep-ph]



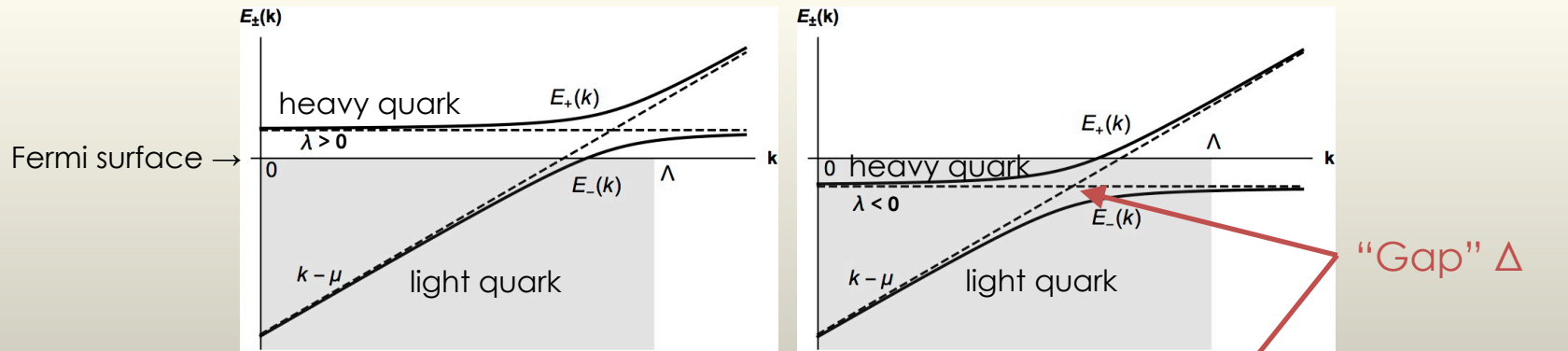
many charm quarks distributed randomly
(No Fermi surface)

Conservation of translational invariance!

$$\begin{aligned}\Psi_v^\dagger(x)\Psi_v(x) &= \sum_i \delta^{(3)}(\vec{x} - \vec{x}_i) \\ &\rightarrow \underline{n_Q \equiv \overline{\sum_i \delta^{(3)}(\vec{x} - \vec{x}_i)}}\end{aligned}$$

2. “Kondo phase” of heavy quark matter

Energy-momentum dispersion & thermodynamic potential



“mixing” mode $E_{\pm}(k) \equiv \frac{1}{2} \left(k + \lambda - \mu \pm \sqrt{(k - \lambda - \mu)^2 + 8N_f |\Delta|^2} \right),$ $k = |\vec{k}|$

“free” mode $\begin{cases} E(k) \equiv E_i = k - \mu & [i = 1, \dots, N_f - 1], \\ \tilde{E}(k) \equiv \tilde{E}_i = -k - \mu & [i = 1, \dots, N_f], \end{cases}$ $\Delta^i = \Delta$

Thanks to Kanazawa-san's comments.

$$\Omega(T, \mu, \lambda; \Delta) = 2N_c \int_0^{\Lambda} f(T, \mu, \lambda; k) \frac{k^2 dk}{2\pi^2} + \frac{8N_f}{G_c} |\Delta|^2 - \lambda n_Q$$

$$f(T, \mu, \lambda; k) \equiv -\beta^{-1} \ln[(1 + e^{-\beta E_+(k)})(1 + e^{-\beta E_-(k)})(1 + e^{-\beta E(k)})^{N_f - 1}]$$

$$\textcircled{1} \quad \frac{\partial}{\partial \Delta^*} \Omega(T, \mu, \lambda; \Delta) = 0 \quad \textcircled{2} \quad -\frac{\partial}{\partial \mu} \Omega(T, \mu, \lambda; \Delta) = n_q \quad \textcircled{3} \quad \frac{\partial}{\partial \lambda} \Omega(T, \mu, \lambda; \Delta) = 0$$

2. “Kondo phase” of heavy quark matter

Gap equation @ $\lambda=0$

S.Y., K. Suzuki, K. Itakura,
arXiv:1604.09229 [hep-ph]

$$\Delta = \frac{1}{2} N_c G_c \int_0^\Lambda \frac{\Delta}{\sqrt{(k - \mu)^2 + 8N_f |\Delta|^2}} \frac{k^2 dk}{2\pi^2} \quad @\lambda=0, T=0$$

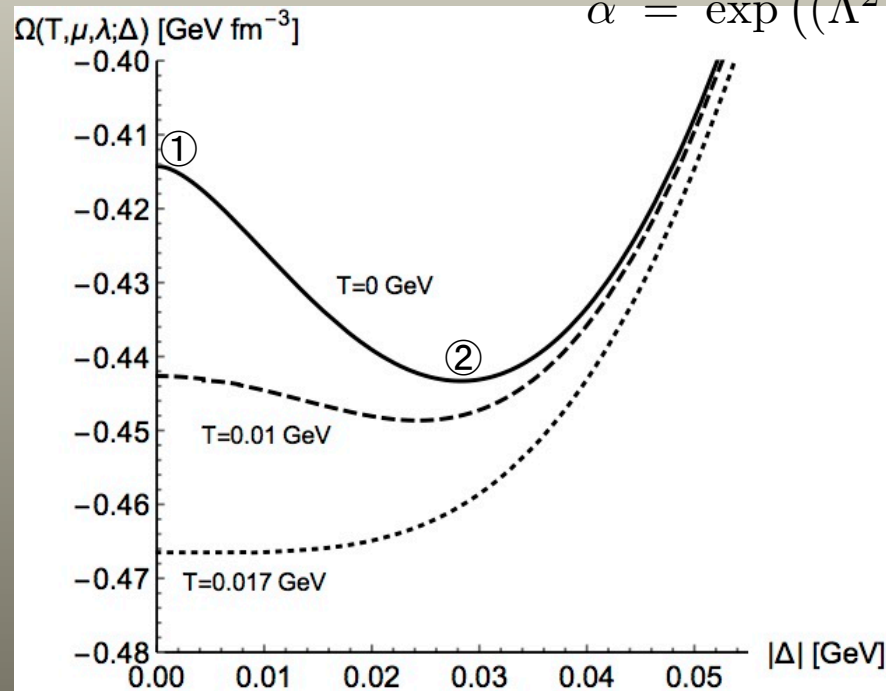
① trivial solution

$$|\Delta| = 0$$

② non-trivial solution

$$|\Delta| \simeq \alpha \sqrt{\frac{\mu(\Lambda - \mu)}{2N_f}} \exp\left(-\frac{2\pi^2}{N_c \mu^2 G_c}\right)$$

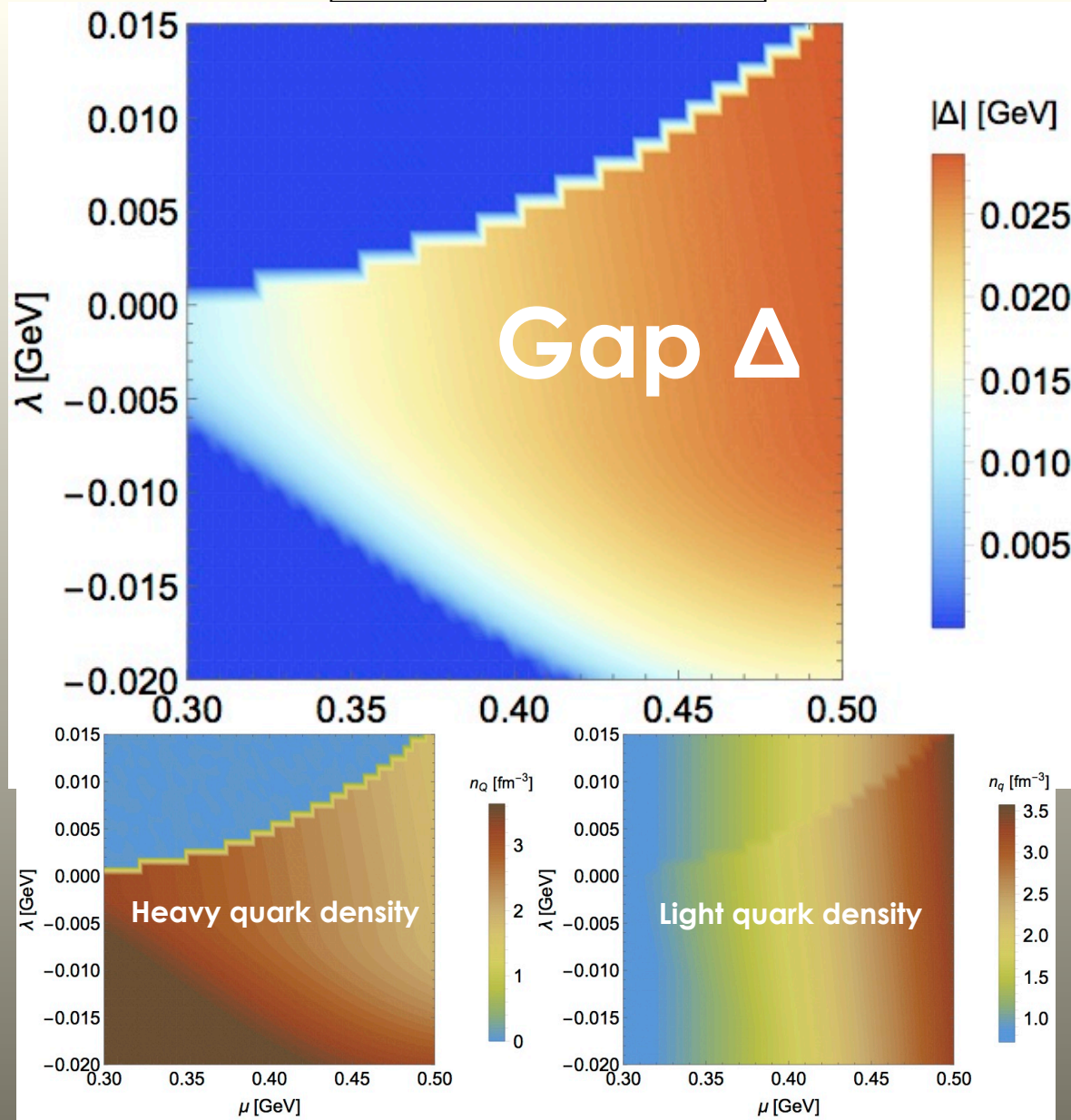
$$\alpha = \exp\left((\Lambda^2 + 2\Lambda\mu - 6\mu^2)/4\mu^2\right)$$



2. “Kondo phase” of heavy quark matter

“Kondo phase” @T=0

S.Y., K. Suzuki, K. Itakura,
arXiv:1604.09229 [hep-ph]

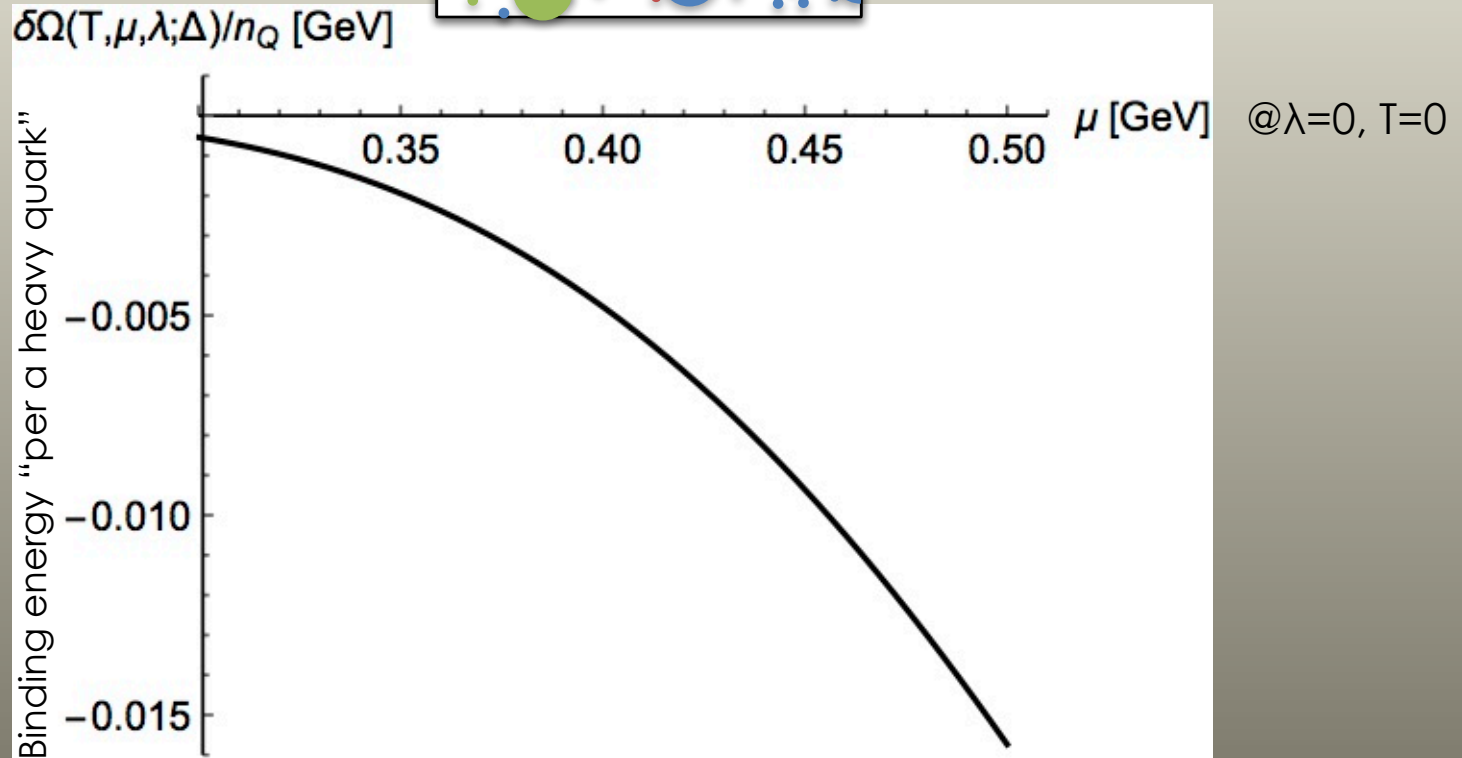
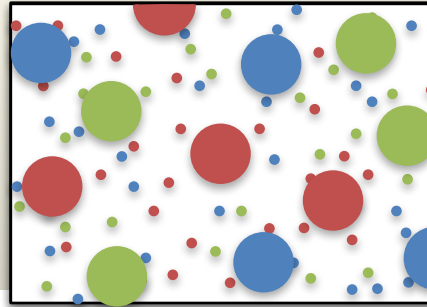


2. “Kondo phase” of heavy quark matter

Energy-gain “per a heavy quark”

S.Y., K. Suzuki, K. Itakura,
arXiv:1604.09229 [hep-ph]

$$\delta\Omega(T, \mu, \lambda; \Delta)/n_Q \quad \text{with} \quad \delta\Omega(T, \mu, \lambda; \Delta) = \Omega(T, \mu, \lambda; \Delta) - \Omega(T, \mu, \lambda; 0)$$



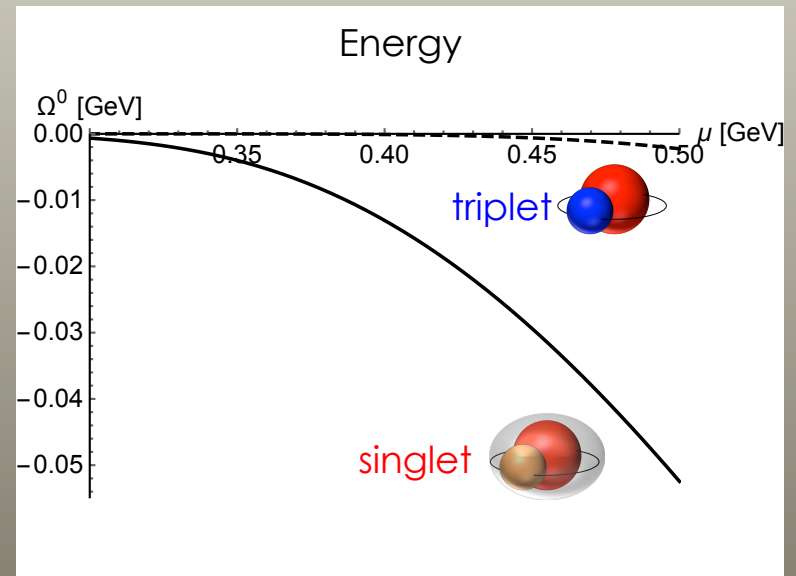
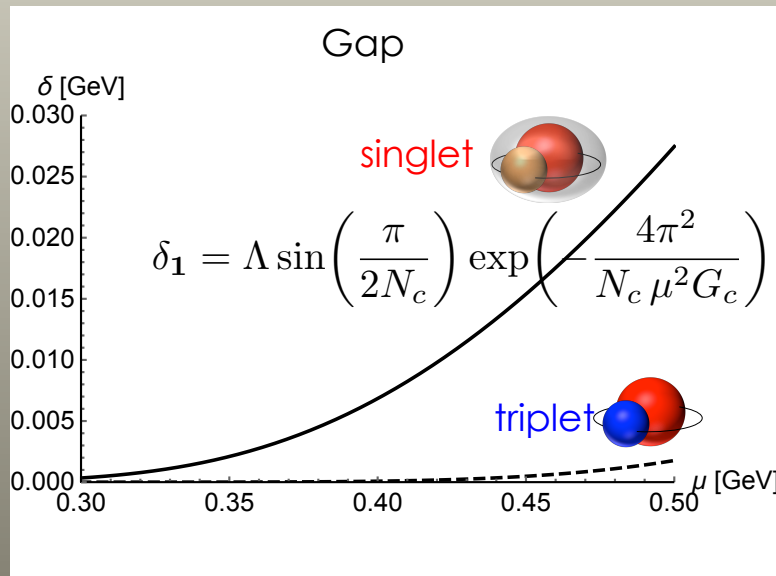
3. Case of single heavy quark

Energy-gain of a heavy quark

S.Y., arXiv:1608.06450 [hep-ph]

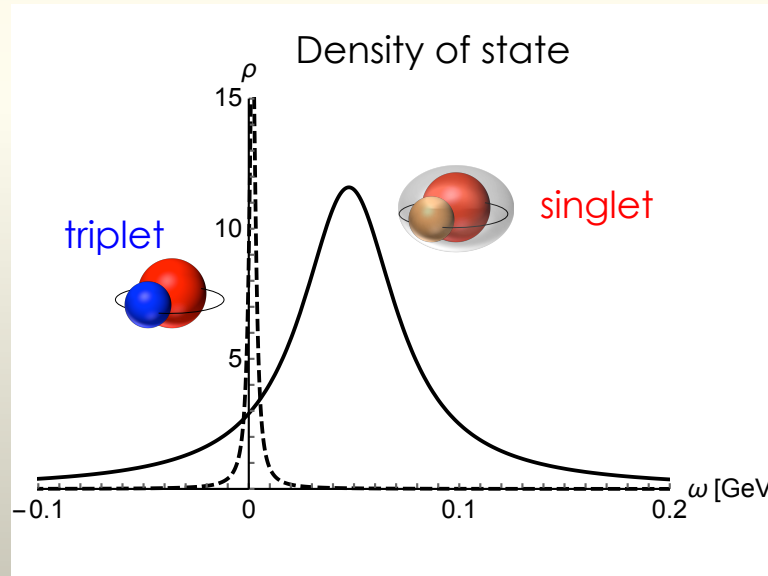
$$\text{Energy: } \Omega_1(\lambda, \Delta_1) = -\frac{1}{\beta} \int_{-\infty}^{+\infty} \ln(1 + e^{-\beta\omega}) 2N_c \rho_1(\omega) d\omega + \frac{8N_f |\Delta_1|^2}{G_c} V - \lambda$$

$$\begin{aligned} \text{Density of state: } \rho_1(\omega) &= -\frac{1}{\pi} \text{Im} \frac{\partial}{\partial \omega} \ln \left(\omega_+ - \lambda - \sum_{\vec{q}} \frac{2N_f |\Delta_1|^2}{\omega_+ + \mu - q} \right) \\ &= \frac{1}{\pi} \frac{\delta_1}{(\omega - \lambda_1)^2 + \delta_1^2} \quad \sum_{\vec{q}} \frac{2N_f |\Delta_1|^2}{\omega_+ + \mu - q} \simeq -\frac{iN_f}{\pi} \mu^2 |\Delta_1|^2 V \equiv -i\delta_1 \\ &\quad \text{(approximation)} \end{aligned}$$

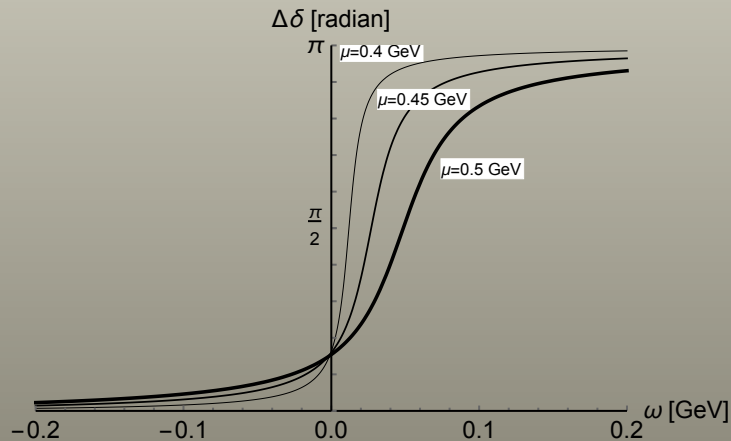


3. Case of single heavy quark

S.Y., arXiv:1608.06450 [hep-ph]

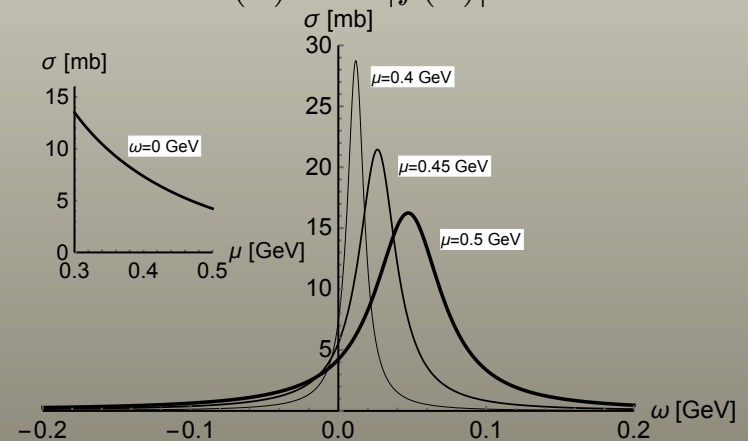


Phase shift: $\Delta\delta(\omega) = \pi \int_{-\Lambda}^{\omega} \rho_1(\omega') d\omega'$



Amplitude: $f(\omega) = e^{i\Delta\delta(\omega)} \sin\Delta\delta(\omega)/k$

Cross section: $\sigma(\omega) = 4\pi |f(\omega)|^2$



Comparable (strongly correlated system)

Mean free path: $\tau \simeq (\sigma n_q / N_f)^{-1}$

$\tau \simeq 1.3 - 0.91$ fm for $\mu = 0.3 - 0.5$ GeV

Mean distance:

$\ell \simeq n_q^{-1/3} = 0.96 - 0.58$ fm

Conclusion of Part 1

① We study the Kondo effect of $\bar{D}s/\bar{D}s^*$ meson

$\bar{D}s/\bar{D}s^*$ meson in nuclear matter

x HQS interaction

= “Kondo resonance”

nucleon in the ground state.

Future works:

1. Application to *realistic* $\bar{D}s/\bar{D}s^*$ meson-nucleon interaction.
2. Application to other heavy hadrons.
3. Observables and reactions for experiments.
4. etc.

Conclusion of Part 2

- ① We study Kondo effect by color exchange for charm quark in quark matter.
- ② The color-current interaction is considered.
- ③ It leads to a nontrivial ground state with mixing between light quark and heavy quark (“Kondo phase”).

Future prospects:

- 1. Realization in QCD phase diagram.
- 2. Observables in experiments.
- 3. Comparison to condensed matter physics. etc.

Related references

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