Impurity problems from 10^{-9} to 10^{13} Kelvin

Shun Uchino

Center for Emergent Matter Science, RIKEN

CEMS

RIKEN

The Sumitomo Foundation

Impurity problems from 10^{-9} to 10^{13} Kelvin

Shun Uchino

Center for Emergent Matter Science, RIKEN

Collaborators

Kazuhiko Kamikado, Toyota

Takuya Kanazawa, RIKEN

Impurities in condensed matter

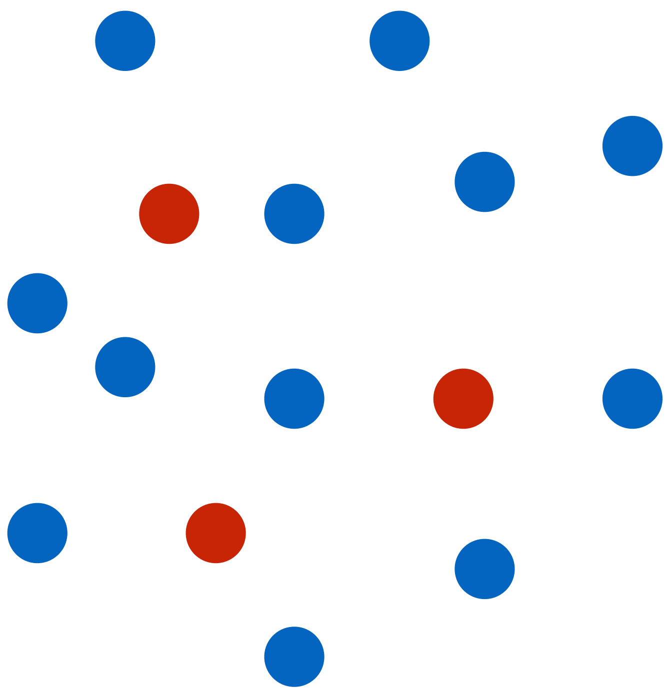
- Inevitable in many cases

  Exceptions: cold atoms, high-mobility semiconductors

- A variety of circumstances

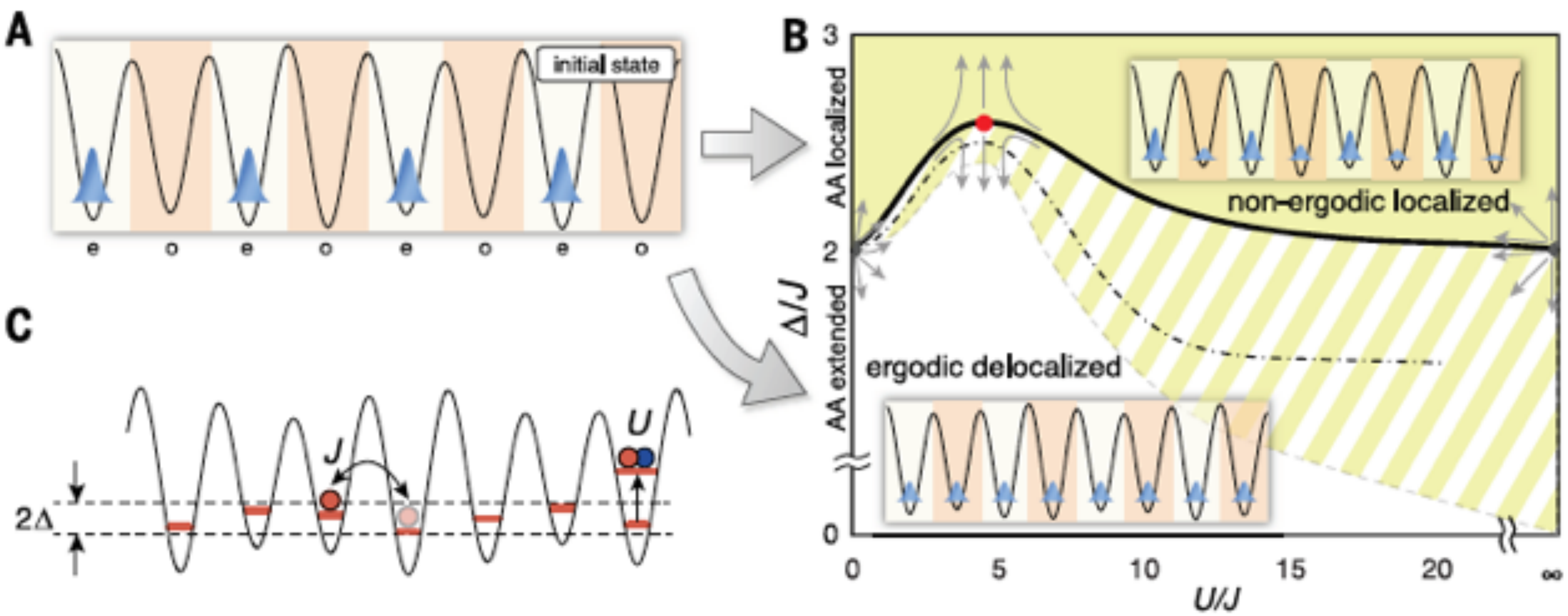
  Moderate density or low enough density

  Mobile or immobile



Relation to Cutting-edge research

- Many-body localization



M. Schreiber et al., Science **349**, 842 (2015).

- Sachdev-Ye-Kitaev model

$$H = \sum_{i,j,k,l} J_{i,j,k,l}\psi_i\psi_j\psi_k\psi_l$$

Sachdev-Ye 1995, Kitaev 2014, Maldacena-Stanford 2016,⋯

Typical example

A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).

Provided

Potential term: $V_{\text{imp}}(\mathbf{x})$

Gaussian: $\langle V_{\text{imp}}(\mathbf{x})\rangle = 0$ $\quad$ $\langle V_{\text{imp}}(\mathbf{x})V_{\text{imp}}(\mathbf{y})\rangle = c\delta(\mathbf{x}-\mathbf{y})$

the bulk one-particle Green's function becomes

$$g(\mathbf{p},\omega_n) = \frac{1}{i\omega_n - \xi_{\mathbf{p}} + \frac{i\mathrm{sgn}(\omega_n)}{2\tau}}$$

collision time: $\tau = \dfrac{1}{2\pi N c}$

⇒ Spectral function becomes Lorenzian

Typical example

A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).

Physical importance: Ohm's law $I = GV$

$$j(\omega) = \sigma(\omega)E(\omega)$$

Drude formula: $\sigma(\omega) = \frac{n}{m}\left(\frac{\tau}{1 - i\omega\tau}\right)$

If τ is infinite,

$$\sigma(\omega) = \frac{\pi n}{m}\delta(\omega)$$

normal gas becomes perfect conductor.

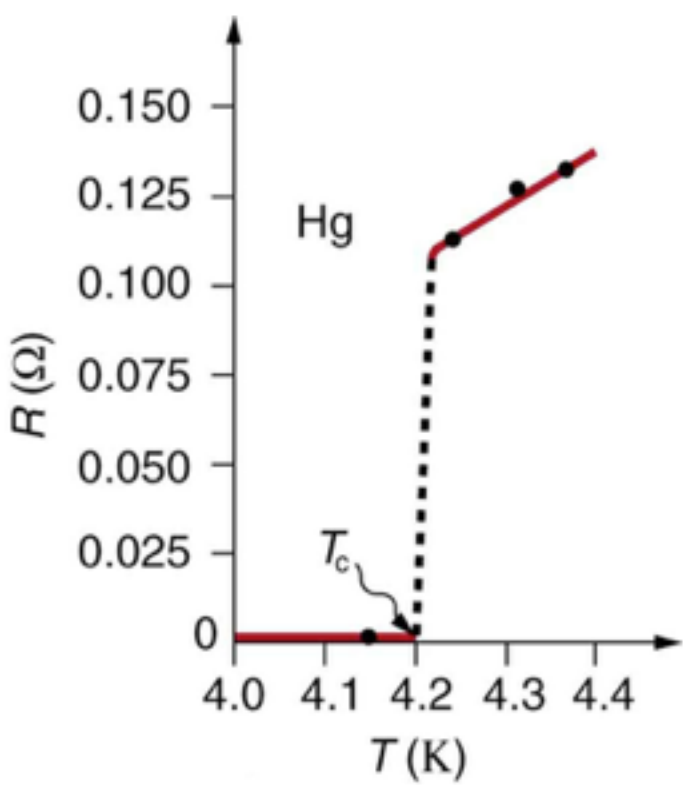
Typical example

A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).

If τ is infinite,

$$\sigma(\omega) = \frac{\pi n}{m}\delta(\omega)$$

we cannot distinguish super from normal



Typical example

A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).

Impurity effect to superconductors

$$V_{\mathrm{imp}} = V_0(x) + \cancel{V_1(x)\sigma \cdot S}$$

(conventional) superconductor is known to be stable (Anderson's theorem)

Dirty: $\xi \gg l$

Clean: $\xi \ll l$

(unconventional superconductor is fragile)

Typical example

A. A. Abrikosov and L. P. Gor'kov, Sov. Phys. JETP **12**, 1243 (1961).

Impurity effect to superconductors

$$V_{\mathrm{imp}} = V_0(x) + V_1(x)\sigma \cdot S$$

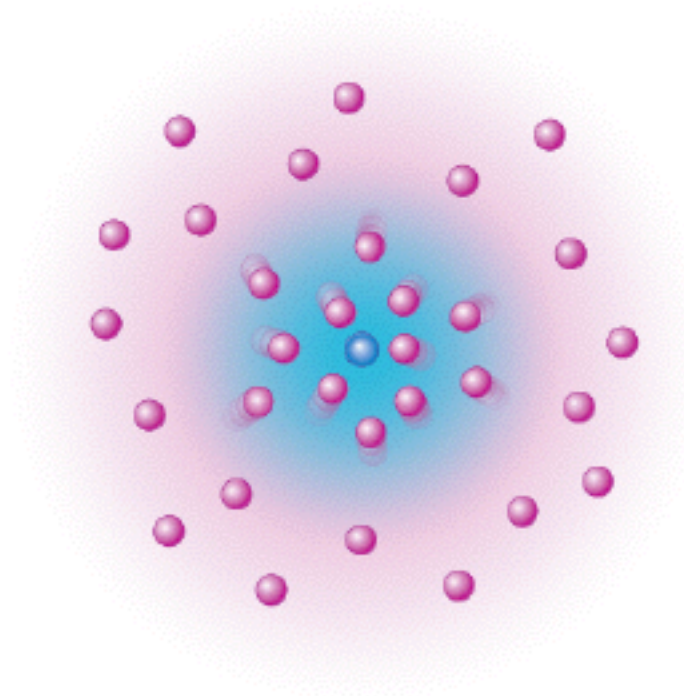
Impurity concentration

Gapless superconductors

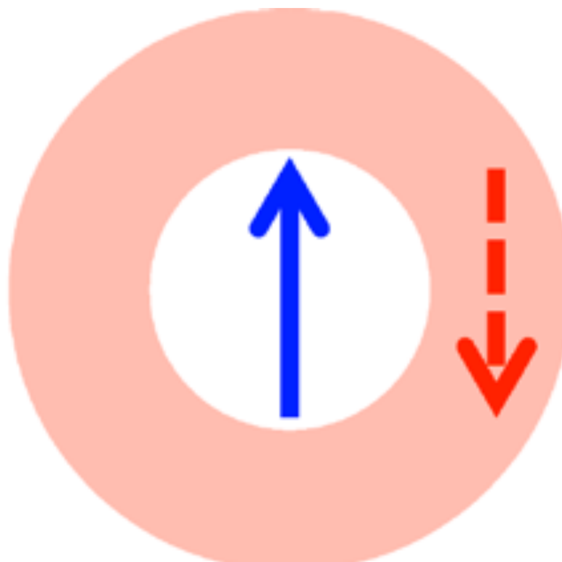
Breakdown of superconductors

Dilute impurity limit

Mobile: Polaron

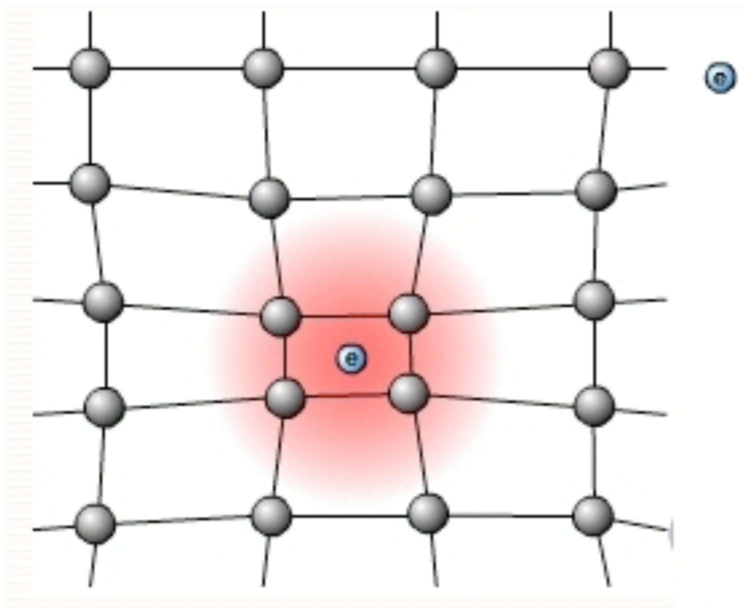


Immobile: Kondo effect



Polaron in solid state

Electron in a phonon bath



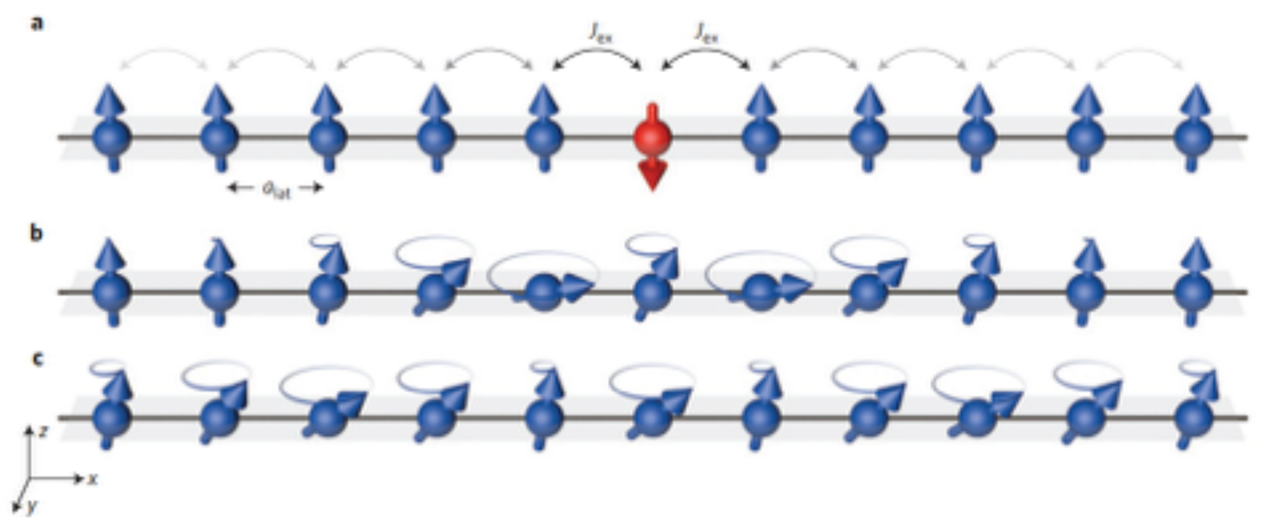
Frohlich Hamiltonian

$$H = \sum_k \frac{k^2}{2m} c_k^\dagger c_k + \sum_q \omega_k a_q^\dagger a_q + \sum_{k,p} v_{k,q} c_k^\dagger c_{k-q} a_q + h.c.$$

$$m \rightarrow m^*$$

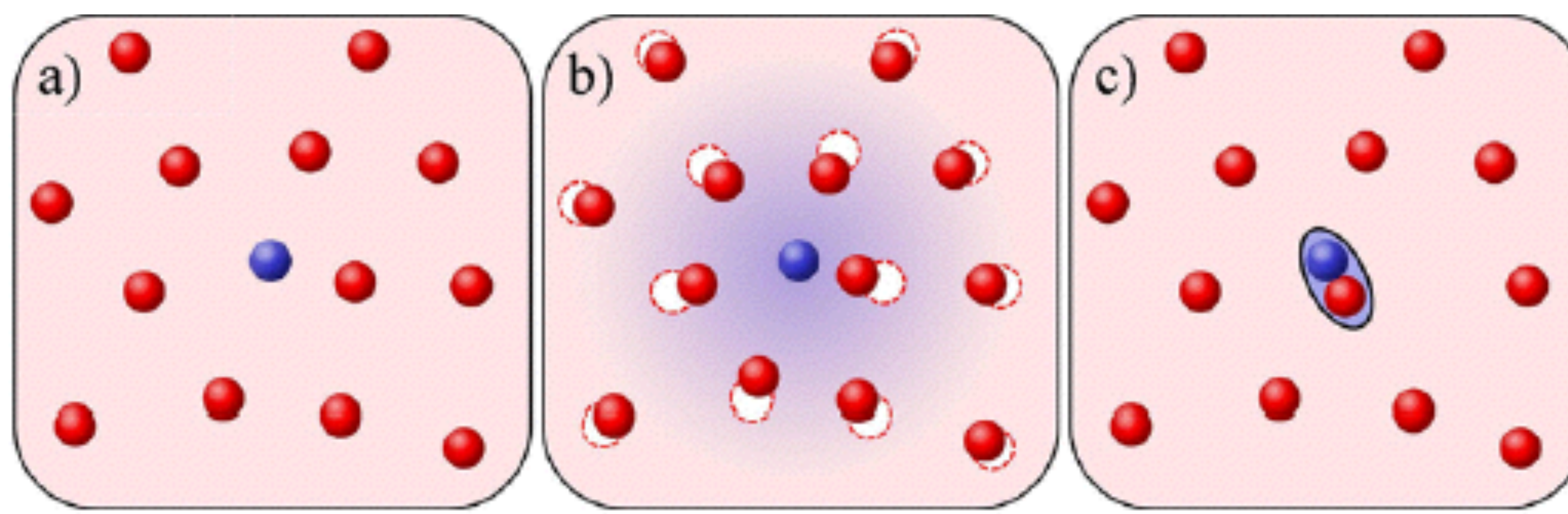
Mobile impurity in cold atoms

1D Magnon dynamics



T. Fukuhara et al., Nature Physics **9**, 235 (2013).

Fermi and Bose Polaron



A. Schirotzek et al., PRL **102**, 230402 (2009);
C. Kohstall et al., Nature **485**, 615 (2012);
M. Koschorreck et al. Nature **485**, 619 (2012);
M. Hu et al., PRL **117**, 055301 (2016);
N. Jorgensen et al., PRL **117**, 055302 (2016),…

Bose polaron

Bose Polaron in cold atoms $\supseteq$ Polaron in solid states

M. Hu et al., PRL **117**, 055301 (2016);
N. Jorgensen et al., PRL **117**, 055302 (2016).

Two-component mixture

$$H = \sum_{i=1,2} \psi_i^\dagger \frac{\nabla^2}{2m_i} \psi_i + g_1 (\psi_1^\dagger \psi_1)^2 + g_2 \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$$

BEC in component1

$$\psi_1 = \sqrt{n_1} + \delta\psi_1$$

$\Downarrow$ Weak coupling

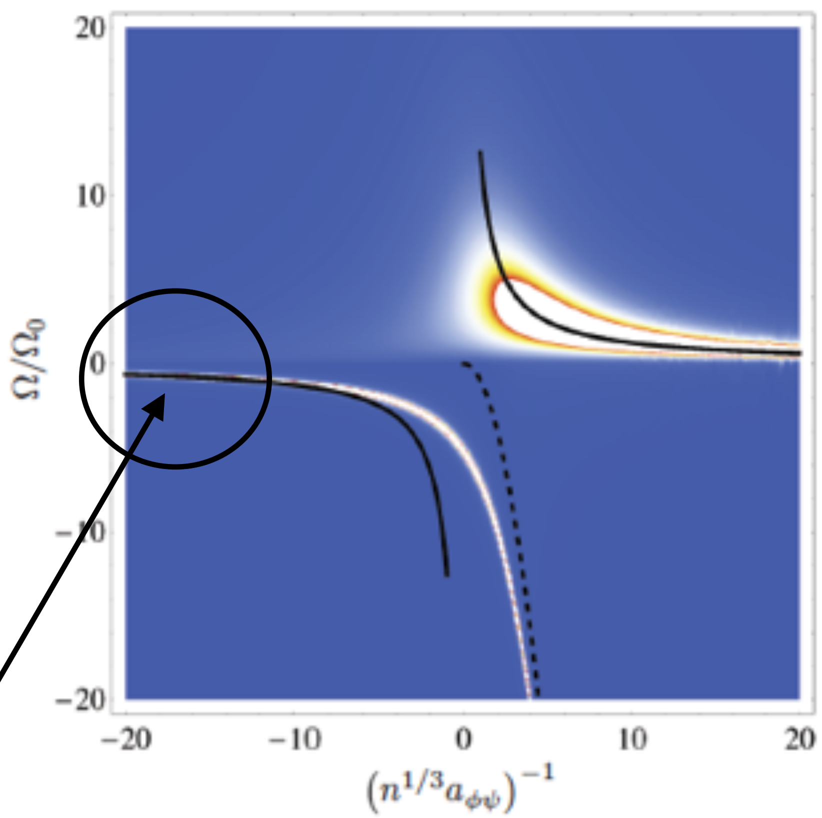
Frohlich Hamiltonian

Bose polaron

Bose Polaron in cold atoms $\supseteq$ Polaron in solid states

M. Hu et al., PRL **117**, 055301 (2016);
N. Jorgensen et al., PRL **117**, 055302 (2016).

Spectral function



Rath and Schmidt, PRA **88**, 053632 (2013).

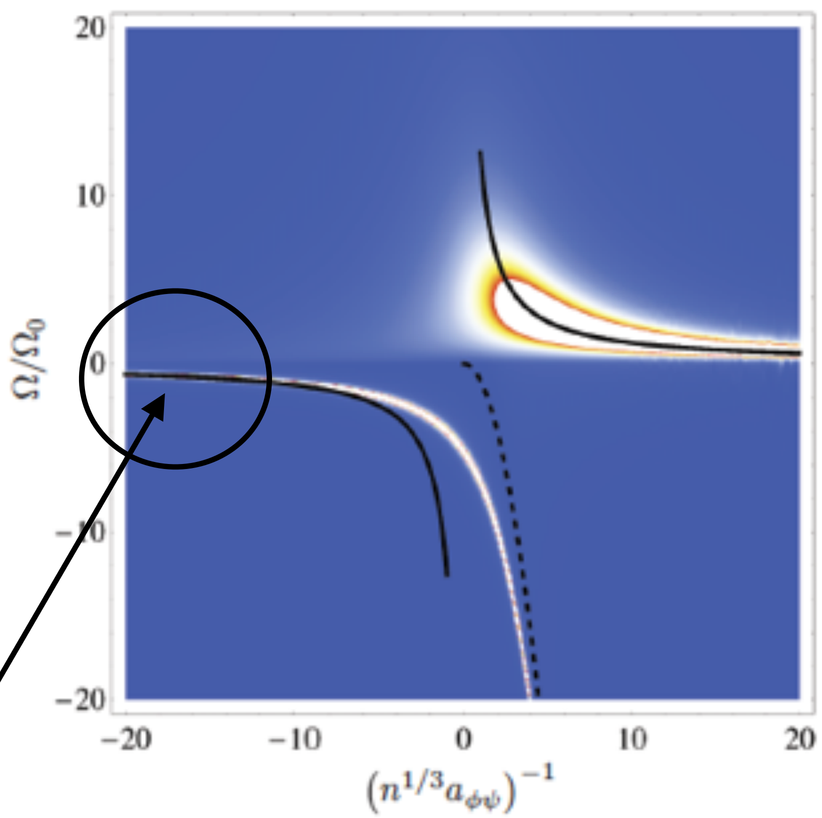
Frolich Hamiltonian is allowed

Bose polaron

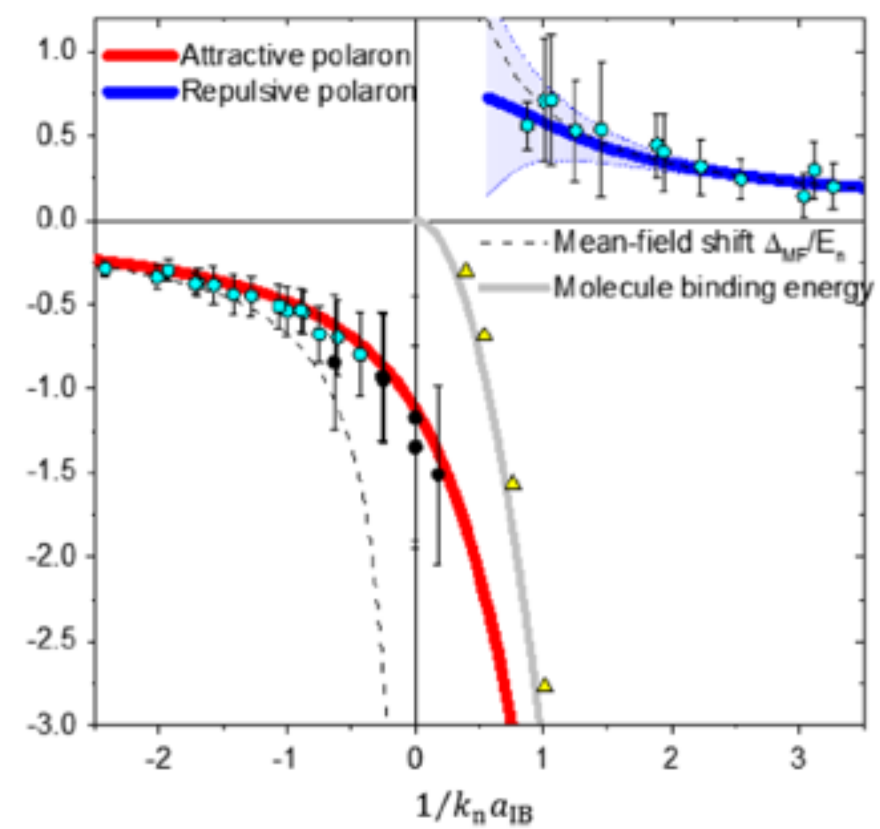
Bose Polaron in cold atoms $\supseteq$ Polaron in solid states

M. Hu et al., PRL **117**, 055301 (2016);
N. Jorgensen et al., PRL **117**, 055302 (2016).

Spectral function



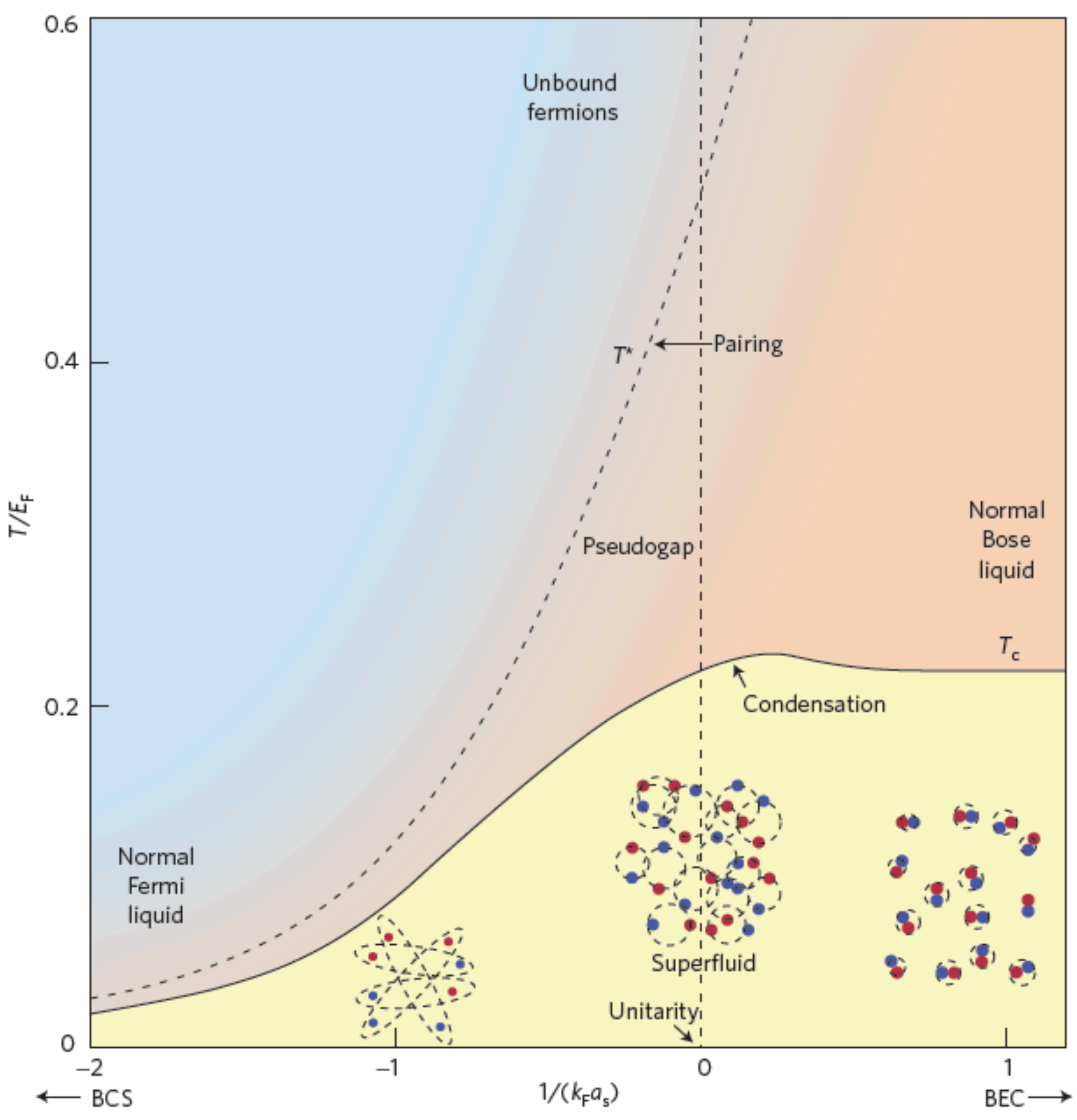
Experiment with RF



Frolich Hamiltonian is allowed

Fermi polaron in cold atoms

BCS-BEC crossover

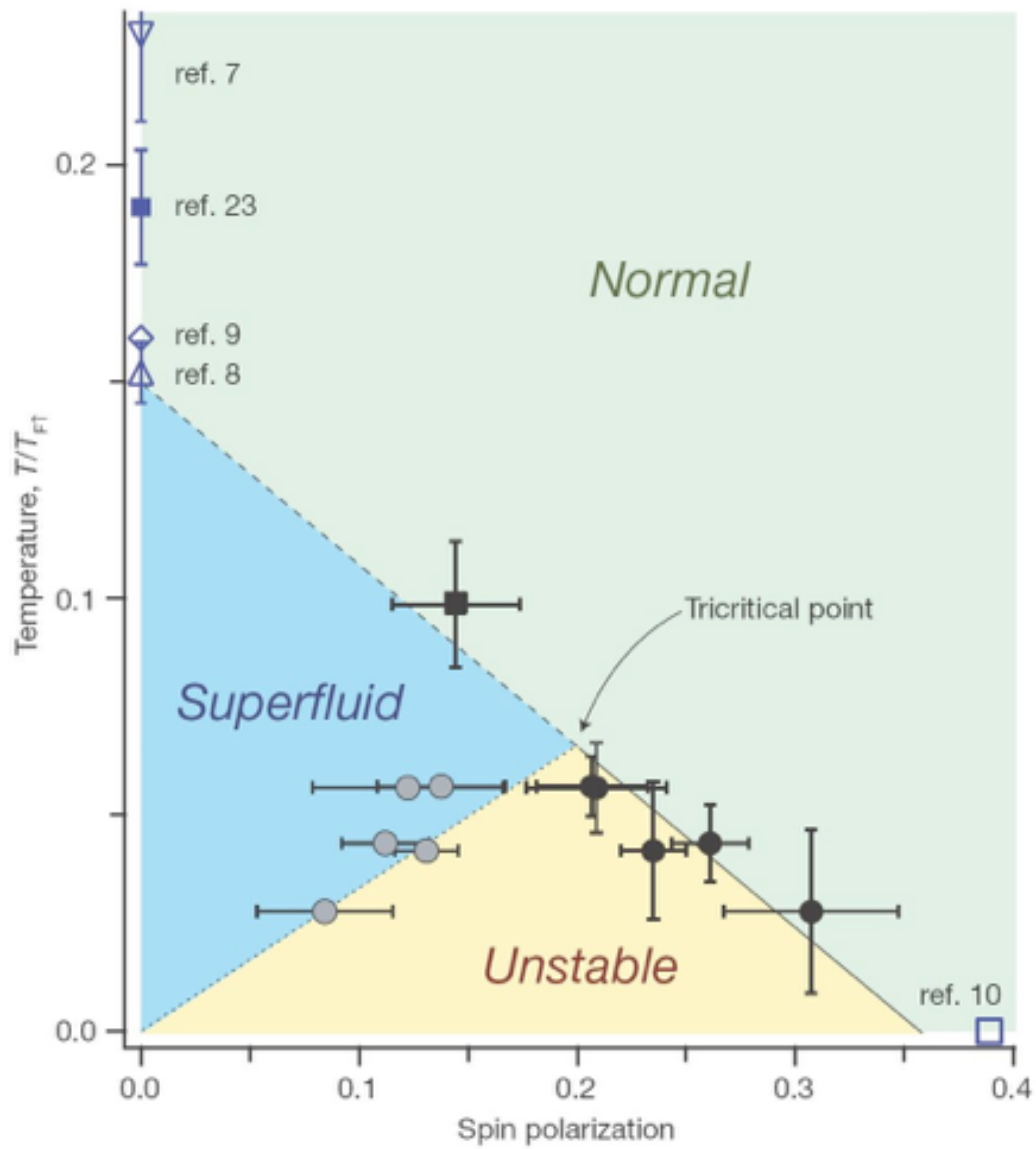


$$n_\uparrow = n_\downarrow$$

M. Randeria, E. Taylor,
Annual Review of Condensed matter physics **5,** 209 (2014).

Fermi polaron in cold atoms

BCS-BEC crossover



$$n_\uparrow \neq n_\downarrow$$

Y. Shin et al., Nature **451**, 689 (2008).

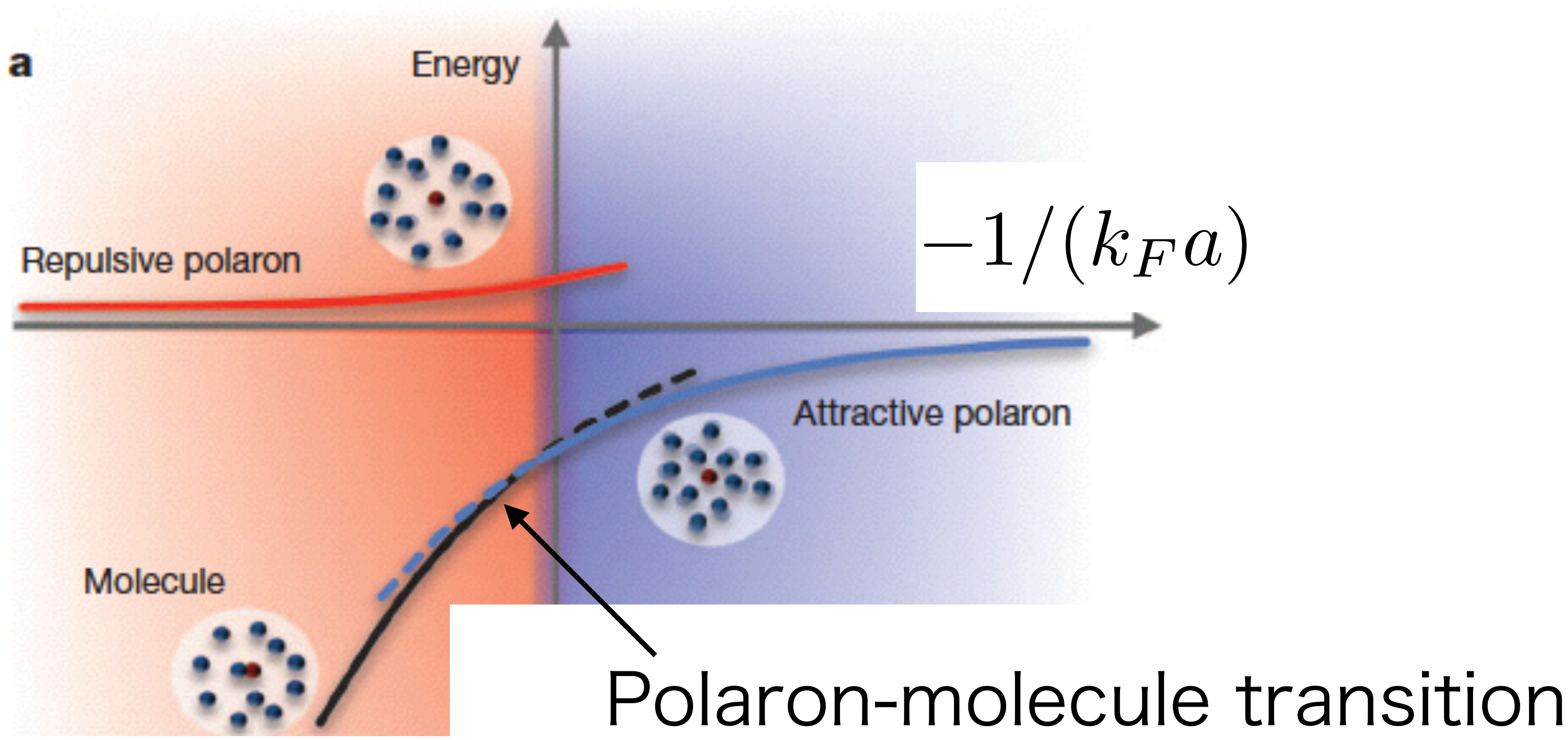
Fermi polaron in cold atoms

what happens if $n_\uparrow \gg n_\downarrow$?

$\downarrow$ particle receives renormalization by a bath of $\uparrow$ particles?

Fermi polaron in cold atoms

M. Koschorreck et al. Nature **485**, 619 (2012);



Polaron problem for validation of quantum many-body technique

Theory

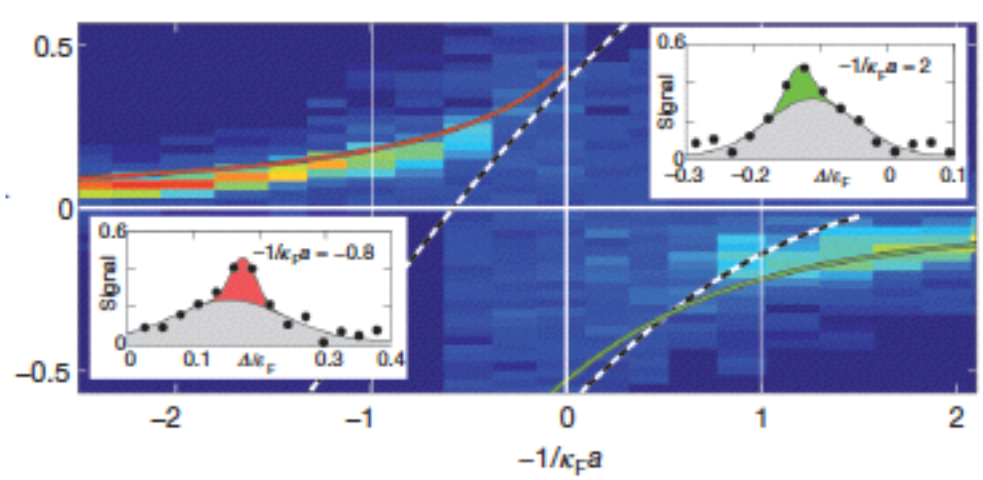
Variational method (Chevy 2006,…)

T-matrix (Combescot 2007,…)

Diagrammatic MC (Prokof'ev-Svistunov 2008,…)

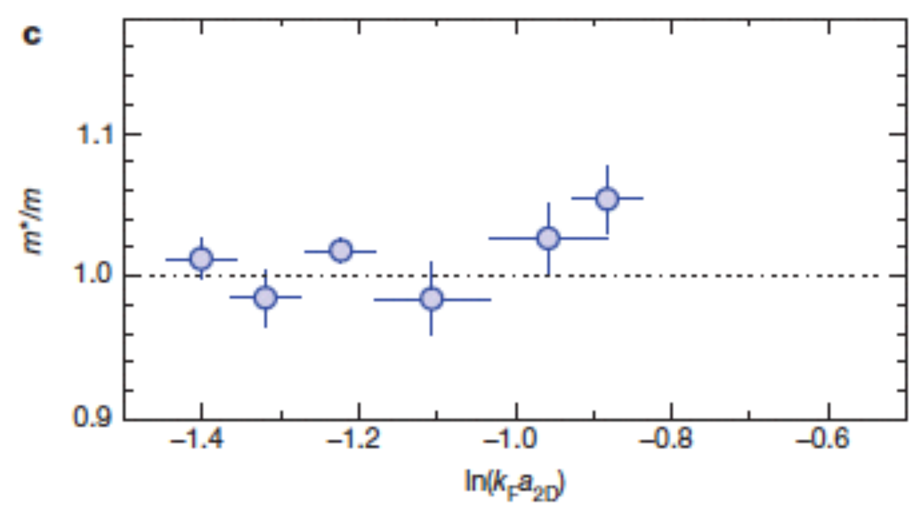
Experiment(rf spectroscopy)

rf response



C. Kohstall et al. Nature **485**, 615 (2012).

Effective mass

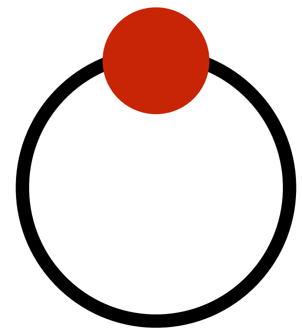


M. Koschorreck et al. Nature **485**, 619 (2012)

Functional renormalization group

Flow equation

$$\boxed{\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} = \frac{1}{2}}$$



C. Wetterich, Phys. Lett. **B301**, 90 (1993).

We use the following ansatz (two-channel model)

$$\boxed{\begin{aligned}\Gamma_k = \int_P \psi_\uparrow^* \left[-ip_0 + \frac{\mathbf{p}^2}{2M_\uparrow} - \mu_\uparrow\right]\psi_\uparrow + \psi_\downarrow^* P_{\downarrow,k}(P)\psi_\downarrow \\ + \phi^* P_{\phi,k}(P)\phi + h(\psi_\uparrow^*\psi_\downarrow^*\phi + \mathrm{H.c.})\end{aligned}}$$

R. Schmidt and T. Enss, PRA **83**, 063620 (2011)

Our approach

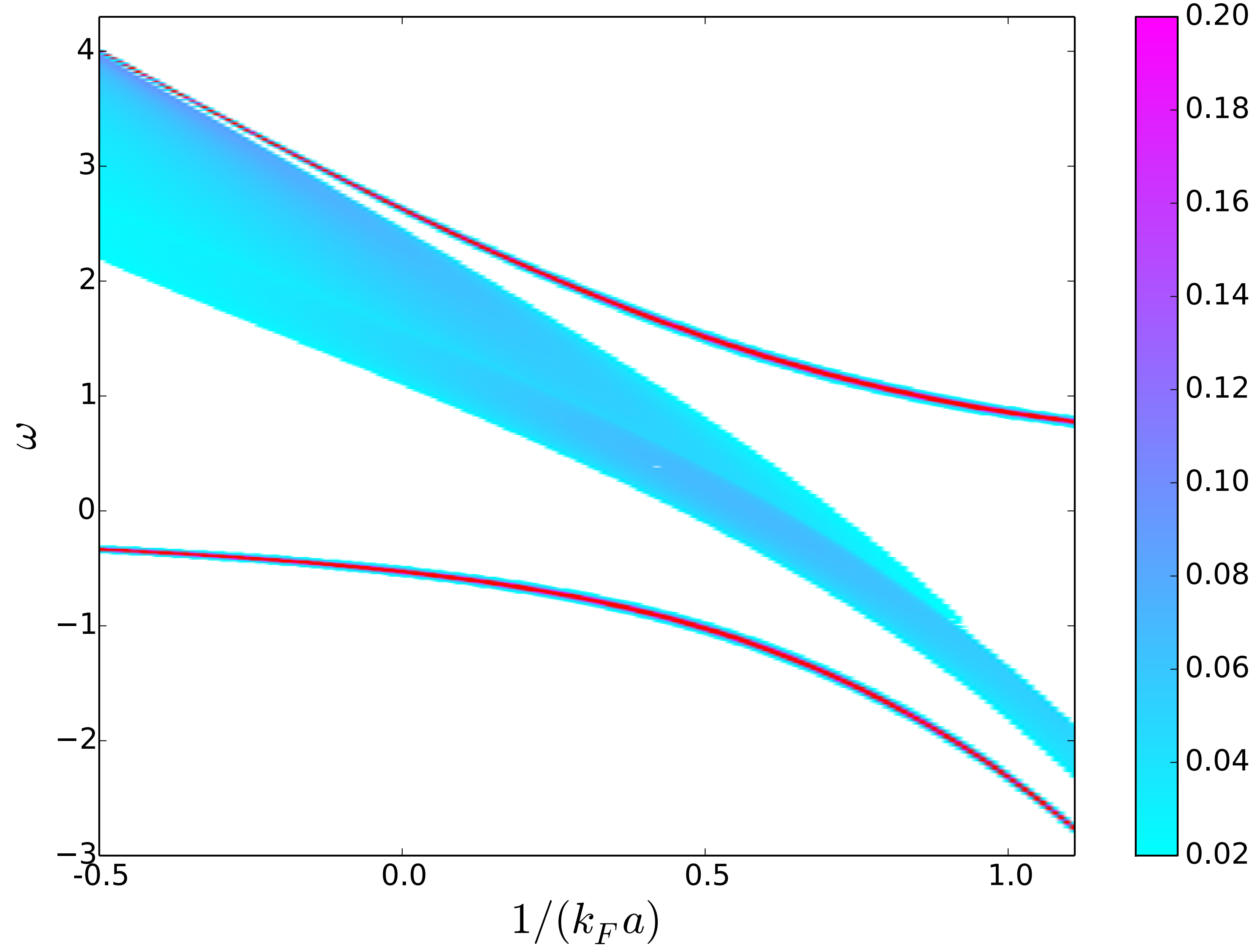
A real time functional renormalization group

$$\partial_t P_{\downarrow}(\omega,p) = \frac{h^2 e^{2t}}{4\pi^2 A_{\phi,k}} \Bigg\{ e^t \Gamma\left[1-2e^{2t}-p^2, 2pe^t, 1-p^2-\frac{\alpha}{\alpha+1}e^{2t}+\frac{m^2_{\phi,k}}{A_{\phi,k}}, \omega, \epsilon\right]$$
$$+\sqrt{1-e^{2t}}\,\theta(-t)\Gamma\left[\frac{1-2e^{2t}-p^2}{\alpha+1}, \frac{2p\sqrt{1-e^{2t}}}{\alpha+1}, \frac{1+p^2+\alpha e^{2t}}{\alpha+1}+\frac{m^2_{\phi,k}}{A_{\phi,k}}, \omega, \epsilon\right]\Bigg\}$$

$$\Gamma[A_1, A_2, A_3, \omega, \epsilon] \equiv \frac{\theta(A_1+|A_2|)}{|A_2|}[\log(|A_2|+A_3-\omega-i\epsilon)$$
$$-\log(\max(0, A_1-|A_2|)+A_3-A_1-\omega-i\epsilon)]$$

K. Kamikado, T. Kanazawa, and SU, arXiv:1606.03721

Spectral function of impurity

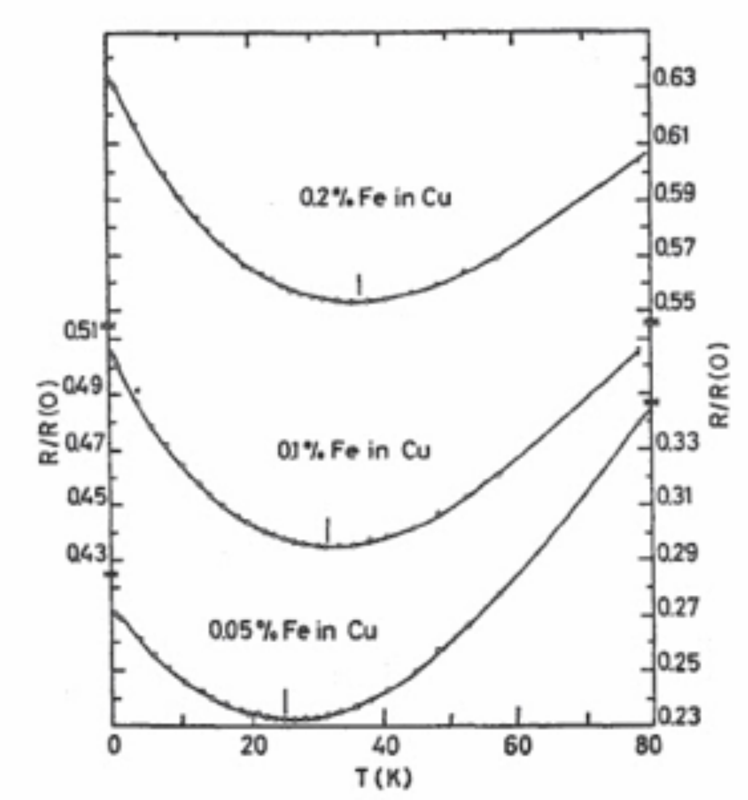


Results are consistent with the other theories and experiments.

Kondo effect

Magnetic impurity in a Fermi sea (metal)

Resistivity minimum at certain temperature



Kondo effect

Anderson model

$$H = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \sum_{\sigma} d^\dagger_\sigma d_\sigma + U n_{d,\uparrow} n_{d,\downarrow} + \sum_{k,\sigma} V_{k,\sigma} d^\dagger_\sigma c_{k,\sigma} + h.c.$$

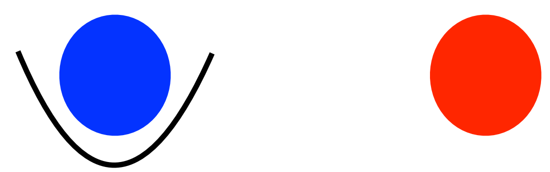
Kondo model

$$H = \sum_{k,\sigma} \epsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + J \sum_{k,k',\sigma} c^\dagger_{k,\sigma} \sigma_{\sigma,\sigma'} c_{k',\sigma'} \cdot \mathbf{S}$$

Kondo model can be derived from Anderson model
as like Heisenberg model from Hubbard model

Kondo effect in cold atoms?

- Not so easy



Spin-flip process forbidden for state-dependent trap

- Proposals for realization

  Bauer, Salomon, Demler (2013), Nishida (2013),
  Nakagawa, Kawakami (2015)

Various Kondo

P. Nozieres and A. Blandin, J. Phys. 41 193 (1980).

Multichannel Kondo model

$$H = \sum_{k,\sigma,l} \epsilon_k c^{\dagger}_{k,\sigma,l} c_{k,\sigma} + J \sum_{k,k',\sigma,l} c^{\dagger}_{k,\sigma,l} \mathbf{s}_{\sigma,\sigma'} c_{k',\sigma',l} \cdot \mathbf{S}$$

RG equation up to J^3

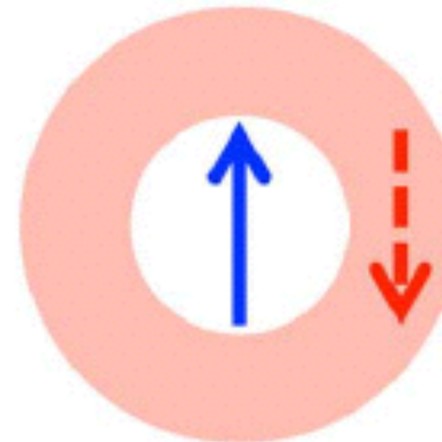
$$\frac{dJ}{d\log D} = -J^2 \left(1 - lJ\right)$$

Various Kondo

P. Nozieres and A. Blandin, J. Phys. 41 193 (1980).

Exact screening Kondo (2s=l)

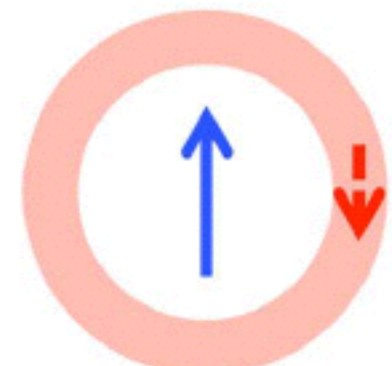
e.g., spin-1/2 and single channel



Underscreening Kondo (2S>l)

Realization in experiment

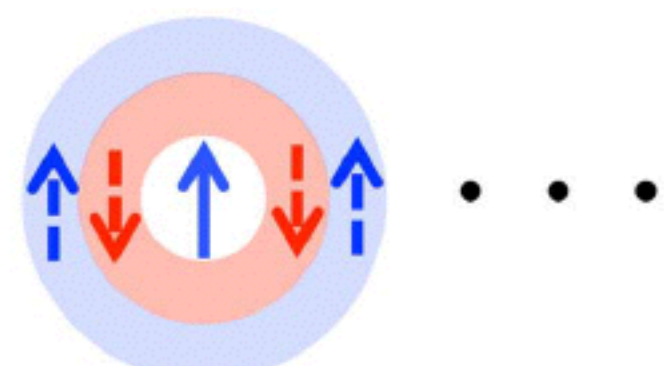
N. Roch et al., PRL **103**, 197202 (2009).



Overscreening Kondo (2s<l)

Realization in experiment

R. Potok et al., Nature **446**, 167 (2007).



Chirality, flavor, spin may be considered as channel degrees of freedom

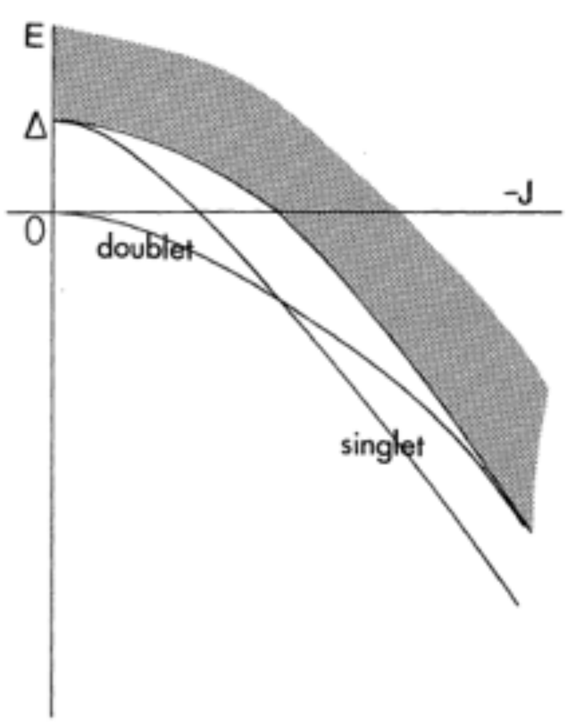
T. Kanazawa and SU, PRD 94, 114005 (2016)

Competition between Kondo effect and superconductivity

- Superconductors try to kill Kondo effect due to absence of states near Fermi level

- Scenario proposed by Shiba by means of classical spin approach

$$J \sum_{k,k'} c^{\dagger}_{k,\sigma} \sigma_{\sigma,\sigma'} c_{k',\sigma'} \cdot \mathbf{S} \rightarrow \tilde{J} \sum_{k,k'} c^{\dagger}_{k,\sigma} \sigma^{z}_{\sigma,\sigma'} c_{k',\sigma'}$$

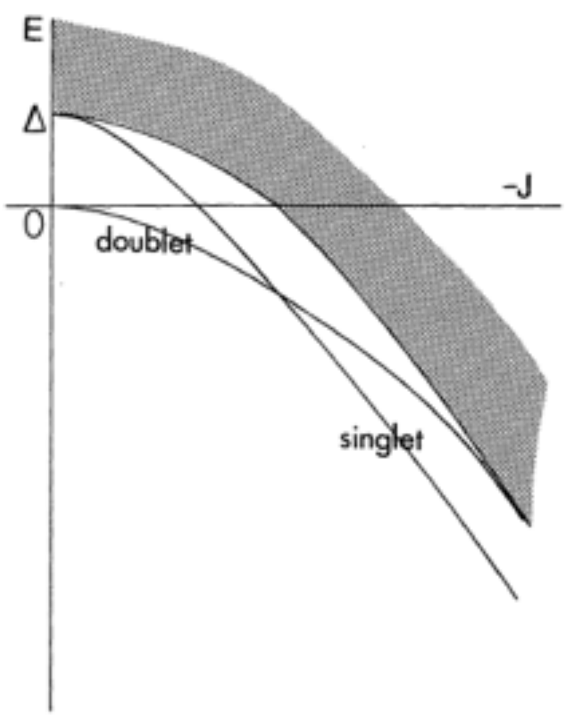


Kondo effect is suppressed when $T_K/\Delta \ll 1$

Competition between Kondo effect and superconductivity

- Similar conclusion can be obtained with perturbation in J (Nagaoka, Matsuura, Soda, Shiba,…)

- NRG analysis concludes that doublet-singlet transition occurs at $T_K/\Delta=0.3$



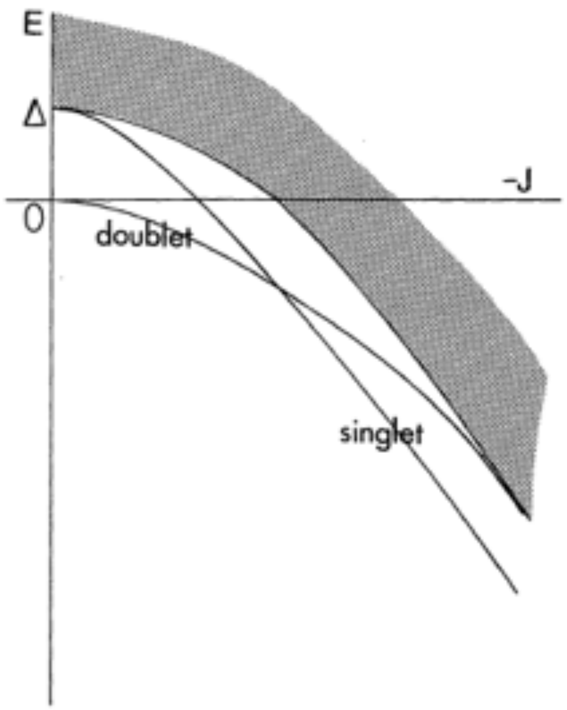
Kondo effect may be difficult in QCD?

T. Kanazawa and SU, PRD 94, 114005 (2016)

Shiba state

- Mid-gap excited state bound to localized impurity

- Emerge for various superconducting systems



Shiba state in color superconductor

T. Kanazawa and SU, PRD 94, 114005 (2016)

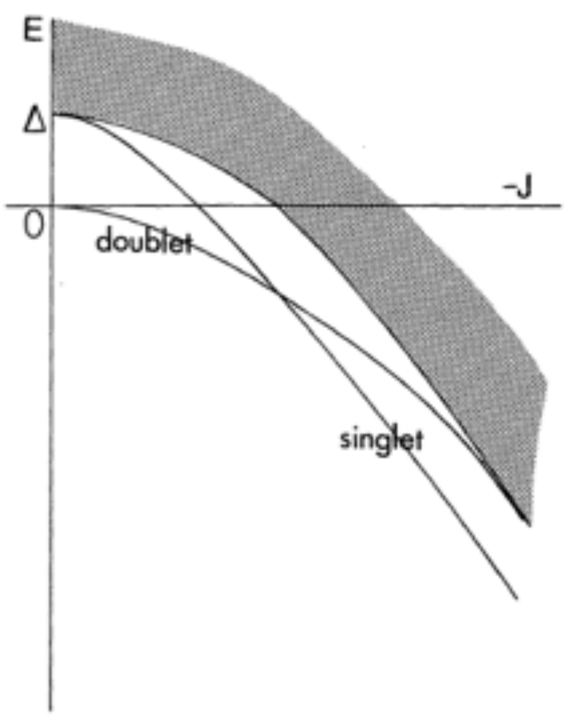
- With classical spin (color) approach, Shiba state can be obtained for a two-flavor color superconducting phase

- Classical spin approach may be subtle to discuss full story.

Possibility of multiple transitions?

R. Zitko and M. Fabrizio, arXiv:1606.07697

What are states to participate the transitions?



Summary

- Impurity problems in various systems
    - cutting-edge research
    - signal
- Polaron in cold atoms
    - Bose polaron
    - Fermi polaron
- Kondo effect in quark matter
    - multichannel Kondo
    - competition between Kondo and superconductor